3D kriging of the ionosphere based on maximum likelihood and restricted maximum likelihood estimation of a non-stationary covariance model

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ABSTRACT

Data assimilation techniques like the successive correction method, optimal interpolation, recursive filters, 3D and 4D variational approaches rely on the correct definition of spatial and temporal covariances, cf. e.g. [1, 2, 4]. For that purpose, the horizontal and vertical correlation lengths are crucial inputs. These parameters are often empirically chosen and wrong specifications might lead to inaccurate estimates of the ionospheric target parameters.

The 3D kriging of the ionospheric electron density is a data assimilation technique that is based on a parametric covariance model of the electron density and allows the estimation of its correlation length parameters from the measurements, cf. [5]. This novel approach provides a best linear unbiased predictor of the electron density $Ne(\vec{x})$ at a coordinate $\vec{x}$ and allows the inclusion of direct electron density measurements, such as in situ or ionosonde derived F2 layer characteristics, as well as linear indirect measurements, e.g ground- and space-based slant TEC (STEC). The parametric covariance model should follow the principle behavior of the ionosphere. Hence, as initial step we define a non-stationary and anisotropic covariance function $Cov_{\vec{\theta}}$ between the electron densities at two coordinates as

$$Cov_{\vec{\theta}}(Ne(\vec{x}_i), Ne(\vec{x}_j)) = \theta_1 \mu(\vec{x}_i)\mu(\vec{x}_j)c_h(h_h; \theta_2, \theta_3)c_v(h_v; \theta_4), \tag{1}$$

where the expectation values of the electron densities $\mu(\vec{x}_i)$ and $\mu(\vec{x}_j)$ are computed by the NeQuick model version 2.0.2, cf. [6]. The product of the expectation values and the scaling factor θ_1 form a non-stationary variance. Further, the horizontal correlation $c_h(h_h; \theta_2, \theta_3)$ with the horizontal distance h_h and the correlation length parameters θ_2, θ_3; and the vertical correlation $c_v(h_v; \theta_4)$ with the vertical distance h_v and the vertical correlation length θ_4 model the anisotropy of the ionosphere in latitude, longitude and height direction.

Since only sparse direct measurements of the electron density are available, the relationship between the covariance of the STEC measurements and the covariance of the electron density is beneficial to determine the unknown parameter vector $\vec{\theta} = (\theta_1, \theta_2, \theta_3, \theta_4)$. If we assume a STEC measurement model along the path s as

$$STEC_s = \int_{\mathbf{s}} Ne(s)ds + \varepsilon_s \tag{2}$$

with a Gaussian and uncorrelated measurement error $\varepsilon_s \sim N(0, \sigma_s^2)$, the covariance of the STEC measurements is related to the covariance of the electron density as

$$Cov_{\vec{\theta}}(STEC_s, STEC_r) = \int_s \int_r Cov_{\vec{\theta}}(Ne(s), Ne(r)) dr ds + Cov(\varepsilon_s, \varepsilon_r). \tag{3}$$

Assuming that the STEC measurements follow a multivariate Gaussian distribution and exploiting Eq. 3, the parameter vector $\vec{\theta}$ of the covariance model can be obtained by a maximum likelihood estimation (MLE). Based on the optimized covariance the electron density at a coordinate is estimated by simple kriging. For more details we refer to [5].

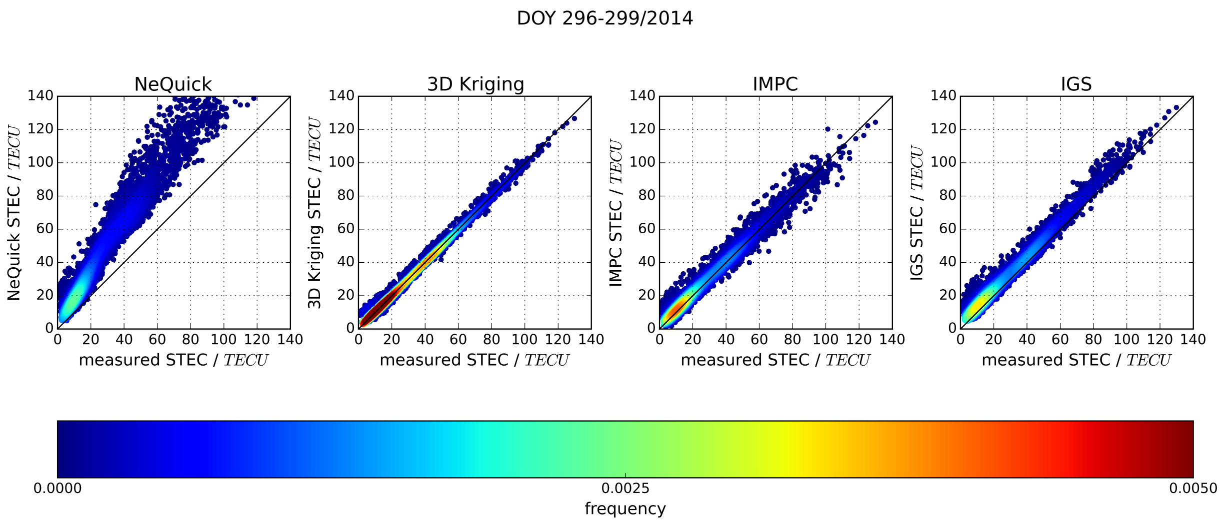

Figure 1. Cross-validation of STEC values estimated by the NeQuick model, 3D kriging, IMPC TEC maps and IGS TEC maps (from left to right) with STEC values from four independent IGS stations in Europe for DOY 296-299/2014.

In this presentation the 3D kriging of the electron density is introduced and its capability to reproduce STEC is validated for two periods covering quiet and perturbed ionospheric conditions. In particular, the STEC measurements of four independent International GNSS Service (IGS) stations in Europe are estimated by kriging and compared to the benchmarks given by the NeQuick model, TEC maps of the DLR's Ionospheric Monitoring and Prediction Center (IMPC) and TEC maps of the IGS. Figure 1 indicates that the kriging allows a promising gain compared to the IMPC STEC of up to 1.1 TECU and 2.4 TECU regarding the mean error and the root mean square error (RMS), respectively.

Moreover, we outline the possible extension of the kriging with calibrated STEC measurements to kriging with uncalibrated STEC measurements. For that purpose, the receiver and satellite related differential code biases (DCB) are added to the STEC measurement model of Eq. (2) and the spatial covariance model of Eq. (1) is extended to a spatial-temporal covariance model. The DCBs are unknown and hence the MLE of the covariance vector $\vec{\theta}$ would suffer from them. However, the DCBs are assumed to be constant over a certain time interval and consequently $\vec{\theta}$ can be estimated by a restricted maximum likelihood estimation (REML) without influence of the DCB bias vector $\vec{\beta}$, cf. [3]. Once $\vec{\theta}$ is optimized, we estimate the DCB bias vector $\vec{\beta}$ with a generalized least squares approach (GLS) including measurements of a sliding window. As an initial validation we calculate the residuals between the calibrated vertical TEC (VTEC) measurements obtained by the combination of REML and GLS and the IGS VTEC. Furthermore, these

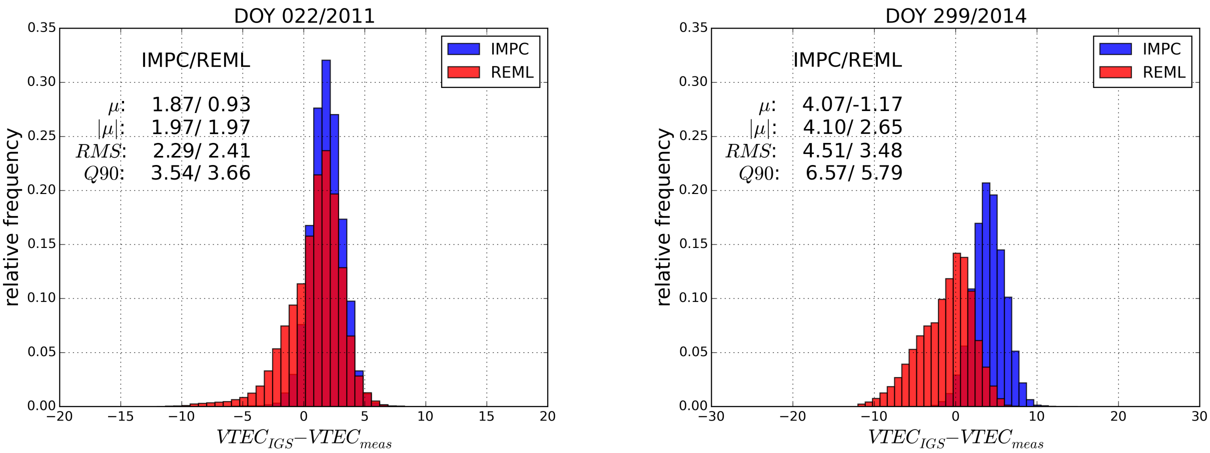

Figure 2. Residuals between the VTEC of the IGS TEC maps and the VTEC estimated by IMPC (blue) and the combination of REML and GLS (red) on the 22th of January 2011 and the 26th of October 2014. Mean (μ), absolute mean ($|\mu|$), RMS and 90 % quantile (Q90) of the residuals are given.

residuals are compared to the residuals between the calibrated VTEC of IMPC and the IGS VTEC for selected days during the quiet and perturbed ionospheric conditions. The results of Figure 2 encourage the follow-up of an approach allowing the usage of uncalibrated STEC measurements and thus an improvement of the measurement error model specification.

Key words: data assimilation, kriging, tomography, multi-sensor, uncalibrated STEC

Acknowledgements We thank the International GNSS Service (IGS) and the Space Physics Interactive Data Resource (SPIDR) for making available high-quality GNSS and geo-related data. Further we express our gratitude to the Aeronomy and Radiopropagation Laboratory of the Abdus Salam International Centre for Theoretical Physics Trieste/Italy providing the NeQuick version 2.0.2 for scientific purposes. This work is financially supported by the Helmholtz Alliance: Remote Sensing and Earth System Dynamics.

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