3D kriging of the ionosphere based on maximum likelihood and restricted maximum likelihood estimation of a non-stationary covariance model

D. Minkwitz*[1], K. G. van den Boogaart[2,3], M. Hoque[1] and T. Gerzen[1]

[1] German Aerospace Center (DLR), Institute of Communications and Navigation, Germany

[2] Helmholtz Institute Freiberg of Resource Technology, Division of Modelling and Evaluation, Germany

[3] University of Mining and Technology Freiberg, Germany

Outline

- Motivation
- Kriging of the ionosphere
  - … with **calibrated** STEC based on MLE
  - … with **relative** STEC based on REML
- Validation scenarios and capability to reproduce STEC
- Conclusions & Outlook

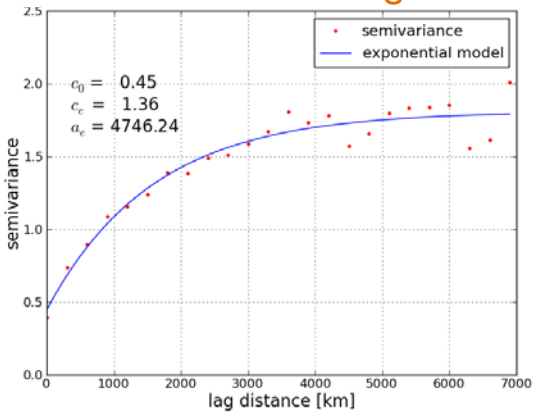
Motivation of 3D kriging

Idea

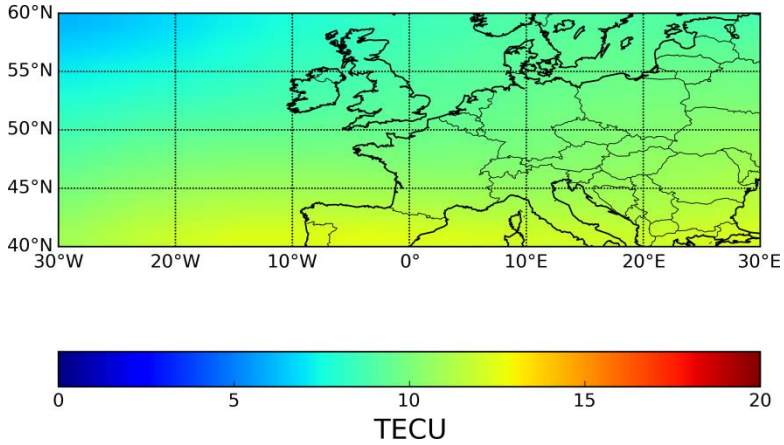
Kriging is already applied to provide ***2D ionospheric estimates***

- for the WAAS system cf. Blanch 2002 & 2003,
- part of the generation of IGS TEC maps, cf. Pérez 2005,
- as well as regional TEC maps, cf. Sayin et al. 2008 & 2010.

Variograms to derive spatial correlation lengths

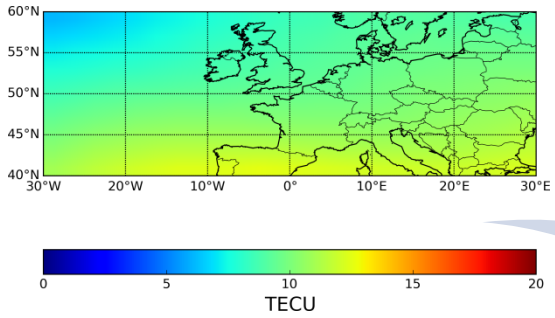


kriging

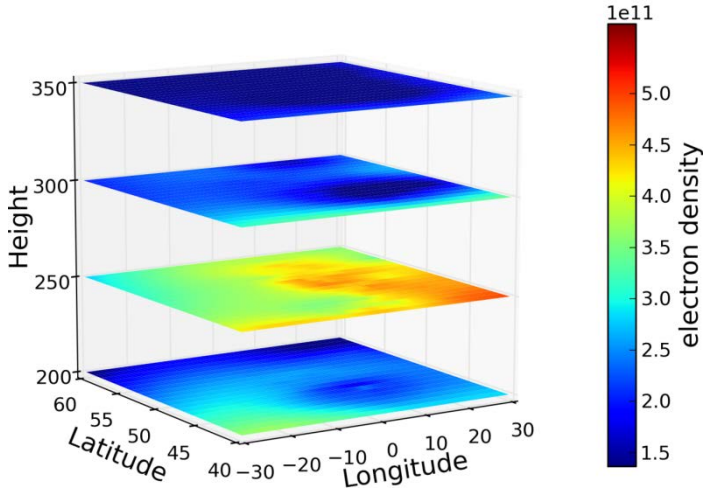


Motivation of 3D kriging

Question



How can the 2D kriging be extended to 3D aiming at the estimation of the electron density?



Kriging with STEC based on MLE

Measurement model & measurement error model

$$STEC_s = \int_s Ne(s)ds + \varepsilon_s \quad \text{with } \varepsilon_s \sim N\left(0, \sigma_s^2(elevation;\ \beta)\right) \text{ and scaling factor } \beta$$

Kriging estimator of electron density $\widehat{Ne}(x)$

$$\widehat{Ne}(\boldsymbol{x}) = \mathbb{E}[Ne(\boldsymbol{x})] + \boldsymbol{\lambda^T} \begin{pmatrix} STEC_1 - \mathbb{E}[STEC_1] \\ \vdots \\ STEC_n - \mathbb{E}[STEC_n] \end{pmatrix}$$, $\mathbb{E}[\dots]$ derived from a background model

BLUE → Optimal weights λ

$$\boldsymbol{\lambda^T} = (\lambda_1, \dots, \lambda_n) = \begin{pmatrix} Cov_{\boldsymbol{\theta}}(Ne(\boldsymbol{x}), STEC_1) \\ \vdots \\ Cov_{\boldsymbol{\theta}}(Ne(\boldsymbol{x}), STEC_n) \end{pmatrix}^T \Sigma^{-1}_{\boldsymbol{\theta},\beta}$$

Covariance matrix of the measurements $\Sigma_{\boldsymbol{\theta},\beta}$ with covariance parameter vector $\boldsymbol{\theta} = (\theta_1, \theta_2, \theta_3, \theta_4)$

Kriging with STEC based on MLE

Set up of covariance model is required

Should reflect basic features of the ionosphere

- Different horizontal and vertical correlation lengths
- Correlation lengths change with time
- Variance of the electron density depends on time and location

Current approach

$$Cov_{\boldsymbol{\theta}}\left(Ne(\boldsymbol{x_1}), Ne(\boldsymbol{x_2})\right) = \theta_1 \cdot \mathbb{E}[Ne(\boldsymbol{x_1})] \cdot \mathbb{E}[Ne(\boldsymbol{x_2})] \cdot c_h(h_h; \theta_2, \theta_3) \cdot c_v(h_v; \theta_4)$$

$$Cov_{\boldsymbol{\theta},\boldsymbol{\beta}}(STEC_s, STEC_r) = \int_s \int_r Cov_{\boldsymbol{\theta}}\left(Ne(s), Ne(r)\right) drds + Cov(\varepsilon_s(\beta), \varepsilon_r(\beta))$$

More details: Minkwitz et al. 2015; doi:10.5194/angeo-33-1071-2015

Kriging with STEC based on MLE

Set up of covariance model is required

Should reflect basic features of the ionosphere

- Different horizontal and vertical correlation lengths
- Correlation lengths change with time
- Variance of the electron density depends on time and location

Current approach

$$Cov_{\boldsymbol{\theta}}\big(Ne(\boldsymbol{x_1}), Ne(\boldsymbol{x_2})\big) = \theta_1 \cdot \mathbb{E}[Ne(\boldsymbol{x_1})] \cdot \mathbb{E}[Ne(\boldsymbol{x_2})] \cdot \boldsymbol{c_h}(\boldsymbol{h_h}; \boldsymbol{\theta_2}, \boldsymbol{\theta_3}) \cdot \boldsymbol{c_v}(\boldsymbol{h_v}; \boldsymbol{\theta_4})$$

$$Cov_{\boldsymbol{\theta},\boldsymbol{\beta}}(STEC_s, STEC_r) = \int_s \int_r Cov_{\boldsymbol{\theta}}\big(Ne(s), Ne(r)\big) drds + Cov(\varepsilon_s(\beta), \varepsilon_r(\beta))$$

More details: Minkwitz et al. 2015; doi:10.5194/angeo-33-1071-2015

Kriging with STEC based on MLE

Set up of covariance model is required

Should reflect basic features of the ionosphere

- Different horizontal and vertical correlation lengths
- Correlation lengths change with time
- Variance of the electron density depends on time and location

Current approach

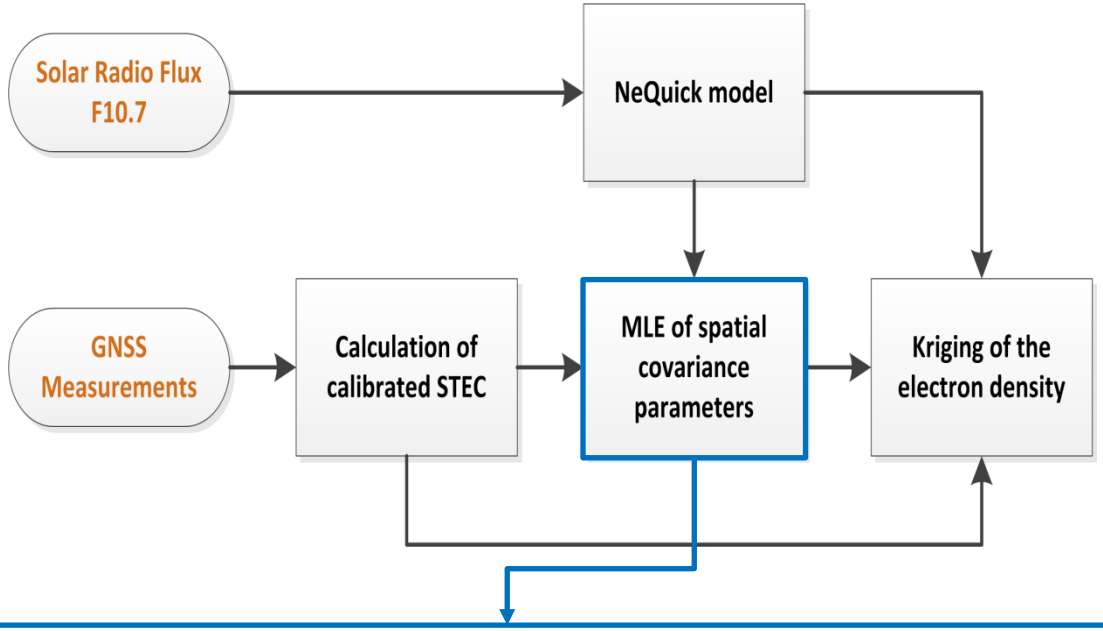
$$Cov_{\boldsymbol{\theta}}\left(Ne(\boldsymbol{x_1}), Ne(\boldsymbol{x_2})\right) = \boldsymbol{\theta_1} \cdot \mathbb{E}[\boldsymbol{Ne}(\boldsymbol{x_1})] \cdot \mathbb{E}[\boldsymbol{Ne}(\boldsymbol{x_2})] \cdot c_h(h_h; \theta_2, \theta_3) \cdot c_v(h_v; \theta_4)$$

$$Cov_{\boldsymbol{\theta},\boldsymbol{\beta}}(STEC_s, STEC_r) = \int_s \int_r Cov_{\boldsymbol{\theta}}\left(Ne(s), Ne(r)\right) dr ds + Cov(\varepsilon_s(\beta), \varepsilon_r(\beta))$$

More details: Minkwitz et al. 2015; doi:10.5194/angeo-33-1071-2015

Kriging with STEC based on MLE

Work flow of the developed 3D kriging



MLE of $\boldsymbol{\theta}, \beta$: Gaussian pdf $f_{\boldsymbol{\theta},\beta}\left(\overrightarrow{STEC}\right) \Longrightarrow \arg\max_{\boldsymbol{\theta},\beta} \ln f_{\vec{\theta},\beta}\left(\overrightarrow{STEC}\right)$

$$\Rightarrow \arg\min_{\boldsymbol{\theta},\beta} \ln\left|\Sigma_{\boldsymbol{\theta},\beta}\right| + (\boldsymbol{STEC} - \mathbb{E}[\boldsymbol{STEC}])^T \Sigma^{-1}_{\vec{\theta},\beta} (\boldsymbol{STEC} - \mathbb{E}[\boldsymbol{STEC}])$$

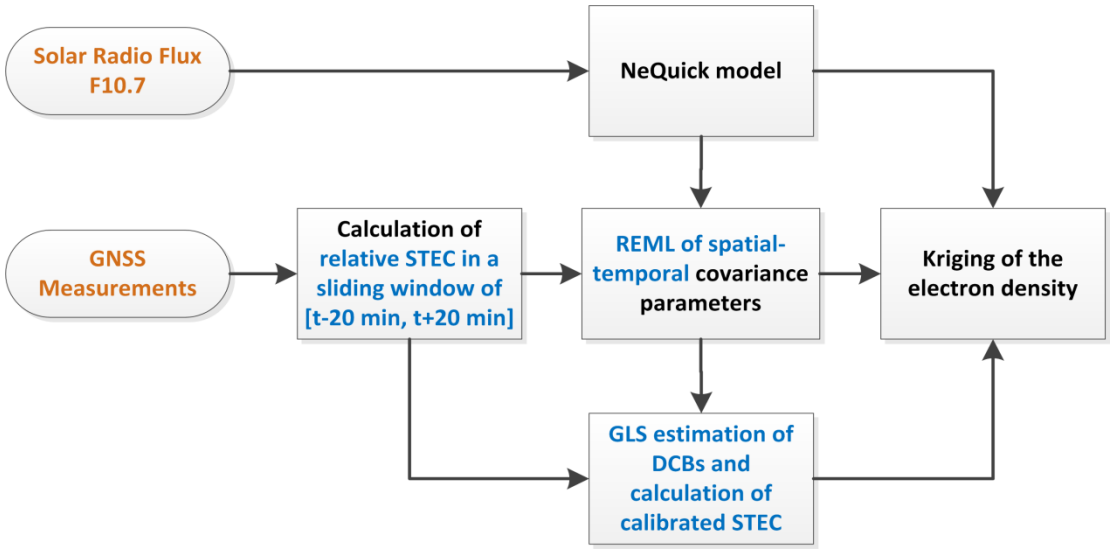
Kriging with relative STEC based on REML

Measurement model

$$STEC_s = STEC_s^{rel} + DCB_{SAT} + DCB_{REC} + \varepsilon_s$$

Differential Code Biases (DCB)

Work flow



Kriging with relative STEC based on REML

REML of the covariance and measurement error model parameters

According to Cressie 1993, p. 93 the covariance parameters can be estimated without influence of the bias vector by

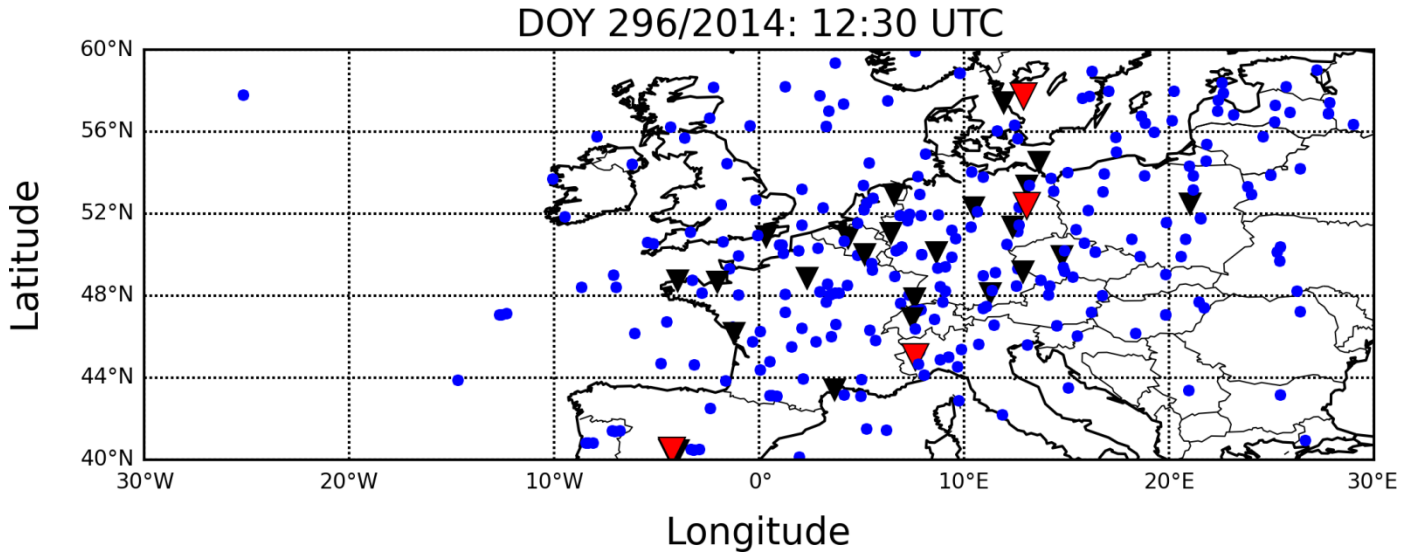
$$\arg\min_{\boldsymbol{\theta},\beta} \ln\left|\Sigma_{\boldsymbol{\theta},\beta}\right| + \ln\left|X^T\Sigma_{\boldsymbol{\theta},\beta}^{-1}X\right| + \left(\boldsymbol{STEC}^{rel} - \mathbb{E}[\boldsymbol{STEC}] - \boldsymbol{Xb}\right)^T \Pi_{\boldsymbol{\theta},\beta}\left(\boldsymbol{STEC}^{rel} - \mathbb{E}[\boldsymbol{STEC}] - \boldsymbol{Xb}\right)$$

$$\Pi_{\boldsymbol{\theta},\beta} = \Sigma_{\boldsymbol{\theta},\beta}^{-1} - \Sigma_{\boldsymbol{\theta},\beta}^{-1}\mathrm{X}\left(X^T\Sigma_{\boldsymbol{\theta},\beta}^{-1}X\right)^{-1}X^T\Sigma_{\boldsymbol{\theta},\beta}^{-1}$$

Estimate the DCB vector b by generalized least squares estimator

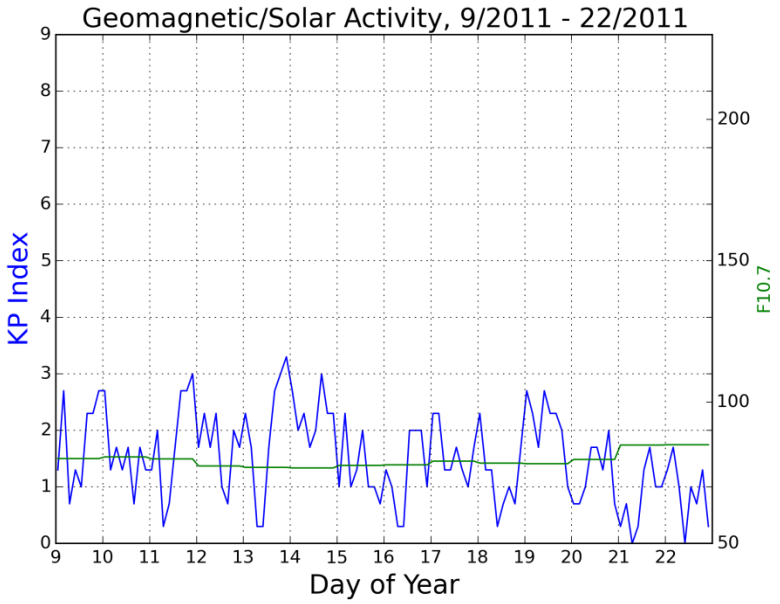
$$\boldsymbol{b} = \left(X^T\Sigma_{\boldsymbol{\theta},\beta}^{-1}X\right)^{-1}X^T\Sigma_{\boldsymbol{\theta},\beta}^{-1}\left(\boldsymbol{STEC}^{rel} - \mathbb{E}[\boldsymbol{STEC}]\right)$$

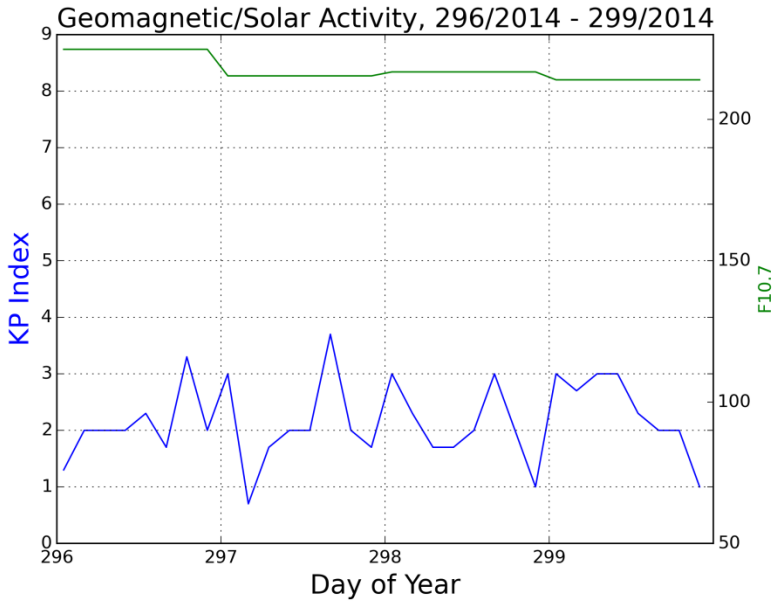
Validation scenario



- Cross-validation: Estimate STEC of red triangle IGS stations without assimilation of the data recorded at these stations
- Comparison of the estimated STEC to STEC obtained by NeQuick, VTEC maps of IGS and DLR's IMPC

Validation scenario

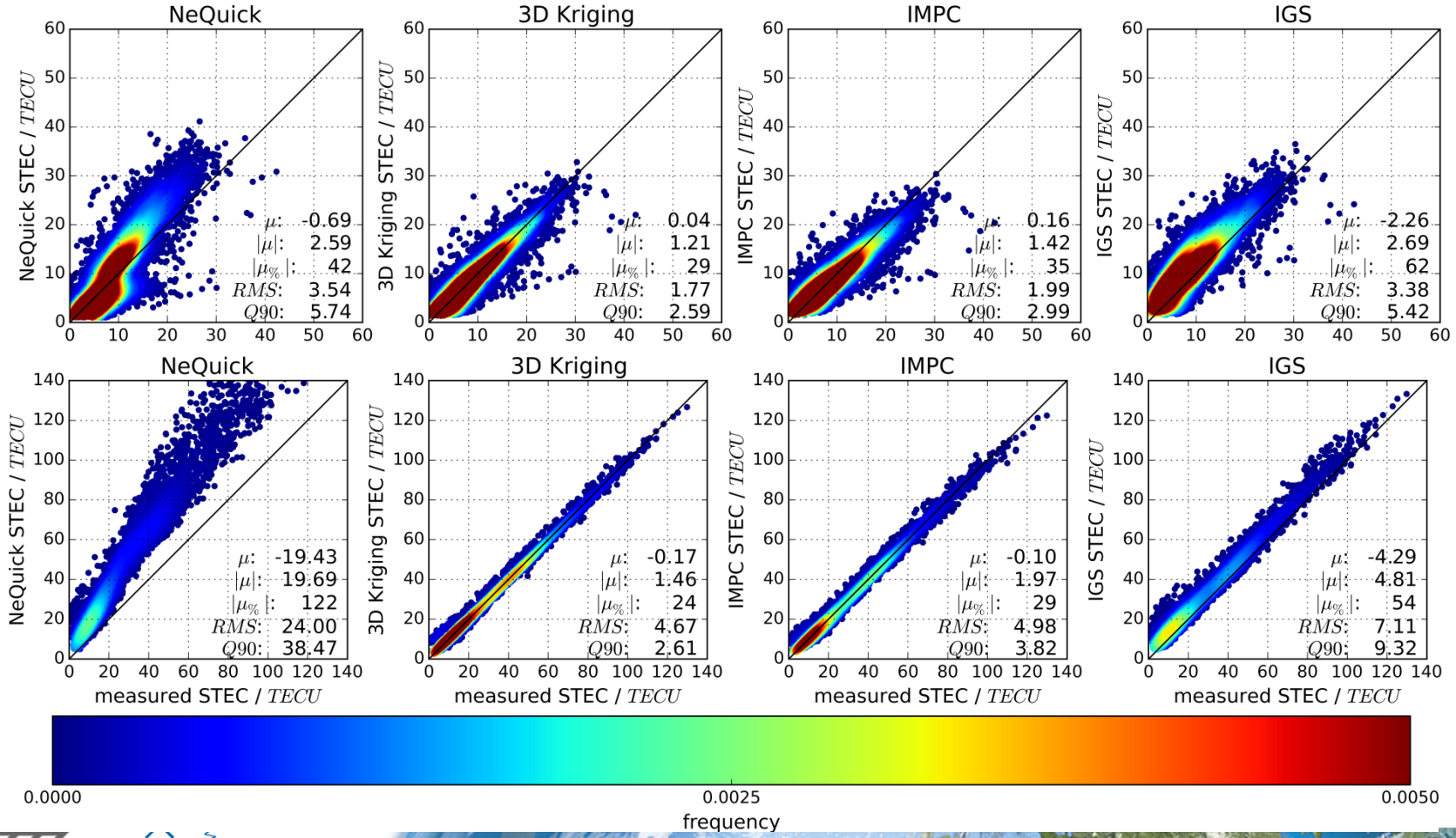




- Two periods: DOY 009-022/2011 & 296-299/2014 representing low and high solar activity

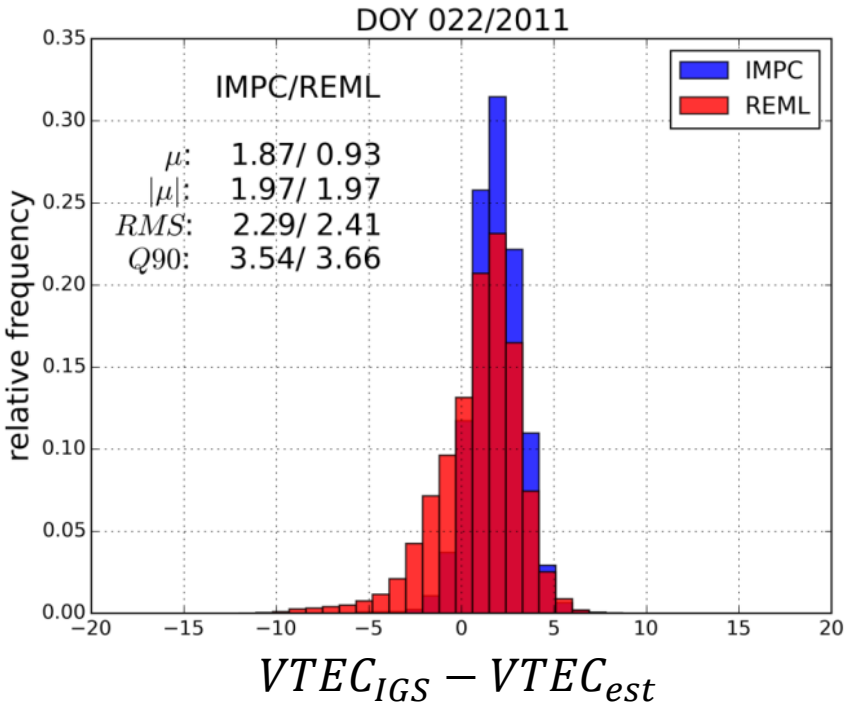
Validation of kriging based on MLE with calibrated STEC

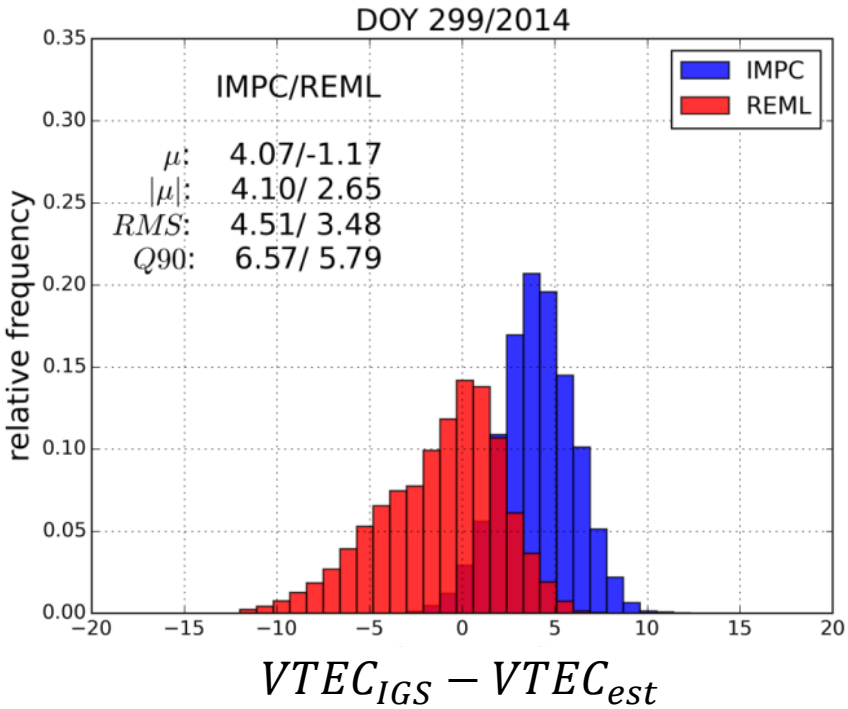
Estimation of STEC: DOY 009-022/2011 (upper); DOY 296-299/2014 (lower)



Validation of kriging based on REML

Initial validation of DCB estimation by comparison of calibrated VTEC estimated by kriging and IMPC with IGS VTEC





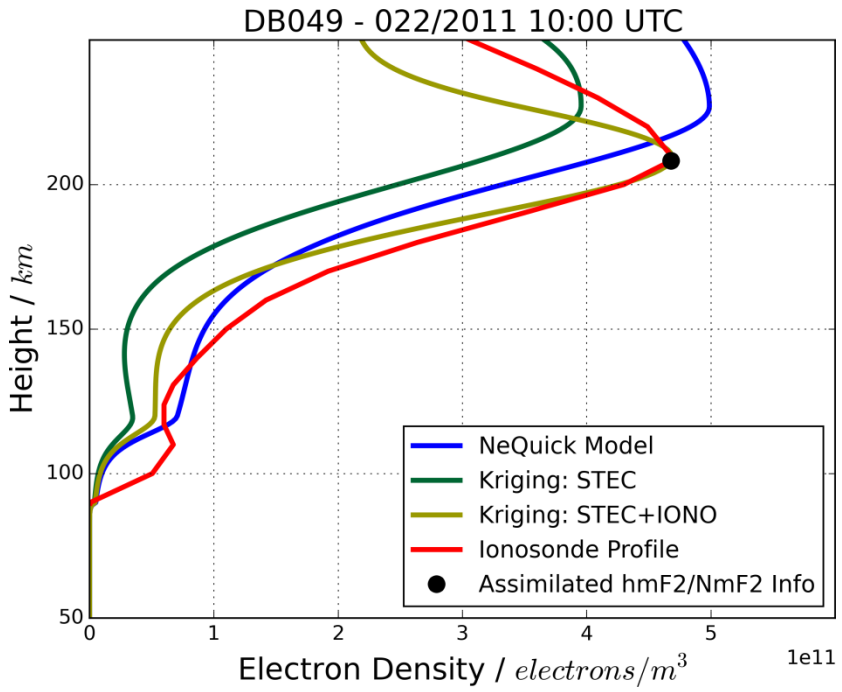
- Mean of difference to IGS VTEC is reduced

Conclusions

- Development of 3D kriging allowing the assimilation of various ionospheric measurement
- Cross-validation of STEC estimation
- 3D kriging approach with relative STEC

Outlook

Assimilate ionosonde measurements to improve layer characteristics in the estimated electron density profiles



Thank you for your attention!

More details:

D. Minkwitz, K. G. van den Boogaart, T. Gerzen, and M. Hoque (2015), Tomography of the ionospheric electron density with geostatistical inversion, Ann. Geophys., 33, 1071-1079, **doi:10.5194/angeo-33-1071-2015**.

Acknowledgement

We express our gratitude to the Aeronomy and Radio Propagation Laboratory of the Abdus Salam International Centre for Theoretical Physics Trieste, Italy, which provided the NeQuick model, version 2.0.2; and to the IGS for making available GNSS data.